Shell-model predictions for $\Lambda\Lambda$ hypernuclei

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Abstract

It is shown how the recent shell-model determination of ΛN spin dependent interaction terms in Λ hypernuclei allows for a reliable deduction of $\Lambda\Lambda$ separation energies in $\Lambda\Lambda$ hypernuclei across the nuclear p shell. Comparison is made with the available data, highlighting $^{11}_{\Lambda\Lambda}$Be and $^{12}_{\Lambda\Lambda}$Be which have been suggested as possible candidates for the KEK-E373 Hida event.

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1. Introduction

The properties of hypernuclei reflect the nature of the underlying baryon-baryon interactions and, thus, provide useful information on the in-medium hyperon-nucleon and hyperon-hyperon interactions. Knowledge of these interactions is required to extrapolate into strange hadronic matter [1] for both finite systems and in bulk, and into neutron stars [2]. Whereas a fair amount of data is available on single-Λ hypernuclei, including production, structure and decay modes [3], little is known definitively on strangeness $\mathcal{S} = -2$ hypernuclei produced to date by recording Ξ^- capture events in emulsion and following their decay sequence [4]. Normally these observed events do not offer unique assignments, except for $^{6}_{\Lambda\Lambda}$He which is the lightest particle-stable $\Lambda\Lambda$ hypernucleus established firmly so far [5]. Numerous $^{6}_{\Lambda\Lambda}$He calculations have been reported since then, including Faddeev [6, 7] and variational [8] $\alpha\Lambda\Lambda$ cluster calculations, as well as VMC [9, 10] and stochastic variational [11, 12] six-body calculations. The separation energy $B_{\Lambda\Lambda}$ of the two Λ's in

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this system exceeds the sum of separation energies B_Λ of each of its Λ's in the single-Λ hypernucleus $^5_\Lambda$He by less than 1 MeV [4]:

$$\Delta B_{\Lambda\Lambda}(_{\Lambda\Lambda}^{6}\text{He}) \equiv B_{\Lambda\Lambda}(_{\Lambda\Lambda}^{6}\text{He}) - 2B_\Lambda(^5_\Lambda\text{He}) = 0.67 \pm 0.17 \text{ MeV}. \qquad (1)$$

Owing to the weakness of the $\Lambda\Lambda$ interaction, it is reasonable to identify $\Delta B_{\Lambda\Lambda}(_{\Lambda\Lambda}^{6}\text{He})$ with the $^1S_0$ $\Lambda\Lambda$ interaction shell-model (SM) matrix element $< V_{\Lambda\Lambda} >_{\text{SM}}$ in the $(1s_\Lambda)^2$ ground state (g.s.) configuration of neighboring hypernuclei. This argument suggests the following estimate for $B_{\Lambda\Lambda}$ in the nuclear p shell:

$$B_{\Lambda\Lambda}^{\text{SM}}(_{\Lambda\Lambda}^{A}Z) = 2\overline{B}_\Lambda(^{A-1}_{\Lambda}Z) + < V_{\Lambda\Lambda} >_{\text{SM}}, \qquad (2)$$

where $\overline{B}_\Lambda(^{A-1}_{\Lambda}Z)$ is the $(2J+1)$-averaged B_Λ in the single-Λ hypernucleus $^{A-1}_{\Lambda}Z$ g.s. doublet, as appropriate to a spin zero $(1s_\Lambda)^2$ configuration of the double-Λ hypernucleus $_{\Lambda\Lambda}^{A}Z$ [7, 8], and $< V_{\Lambda\Lambda} >_{\text{SM}} = 0.67 \pm 0.17$ MeV.[1]

The remarkably simple SM estimate (2), for nuclear core spin $J_c \neq 0$, requires the knowledge of B_Λ for both the g.s. as well as its Λ hypernuclear doublet partner which normally is the first excited state and which experimentally is not always known. However, recent advances in Λ hypernuclear spectroscopy [14] made it possible to derive the spin-dependent ΛN interaction matrix elements [15] directly from γ ray measurements, and thus to relate $\overline{B}_\Lambda(^{A-1}_{\Lambda}Z)$ to experimentally determined g.s. separation energies $B_\Lambda^{\text{exp}}(^{A-1}_{\Lambda}Z_{\text{g.s.}})$. In this Letter, we discuss briefly the g.s. spectroscopy of single-Λ hypernuclei and show how to derive the $\overline{B}_\Lambda$ input to Eq. (2) in order to predict g.s. $B_{\Lambda\Lambda}$ values in $\Lambda\Lambda$ hypernuclei. These predictions work remarkably well for $_{\Lambda\Lambda}^{10}\text{Be}$ and $_{\Lambda\Lambda}^{13}\text{B}$ which are the only emulsion events assigned with high degree of consistency so far beyond $_{\Lambda\Lambda}^{6}\text{He}$. We present predictions for $_{\Lambda\Lambda}^{11}\text{Be}$, $_{\Lambda\Lambda}^{12}\text{Be}$ and $_{\Lambda\Lambda}^{12}\text{B}$ to confront several recently reported alternative interpretations to the KEK-E176 event generally accepted as $_{\Lambda\Lambda}^{13}\text{B}$ [16], and particularly to confront the very recent KEK-E373 HIDA event [4]. Our SM prediction for $_{\Lambda\Lambda}^{11}\text{Be}$ agrees with a recent CM prediction for $_{\Lambda\Lambda}^{11}\text{Be}$ treated as a five-body $\alpha\alpha n\Lambda\Lambda$ cluster [13]. Our SM prediction for $_{\Lambda\Lambda}^{12}\text{Be}$ is without competition, resulting in a definitive statement that neither $_{\Lambda\Lambda}^{11}\text{Be}$ nor $_{\Lambda\Lambda}^{12}\text{Be}$ provides satisfactory agreement with the HIDA emulsion event [4].

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[1]In cluster model (CM) calculations [13], $< V_{\Lambda\Lambda} >_{\text{CM}} \equiv B_{\Lambda\Lambda}(V_{\Lambda\Lambda} \neq 0) - B_{\Lambda\Lambda}(V_{\Lambda\Lambda} = 0)$ assumes similar values: 0.54 MeV for $_{\Lambda\Lambda}^{6}\text{He}$ and 0.53 MeV for $_{\Lambda\Lambda}^{10}\text{Be}$.

2. Input from Λ hypernuclear shell model

The ΛN effective interaction for a $1s_\Lambda$ orbital in the nuclear p shell is given by [17]

$$V_{\Lambda N}(r) = V_0(r) + V_\sigma(r)\,\vec{s}_N \cdot \vec{s}_\Lambda + V_\Lambda(r)\,\vec{l}_{N\Lambda} \cdot \vec{s}_\Lambda + V_N(r)\,\vec{l}_{N\Lambda} \cdot \vec{s}_N + V_T(r)\,S_{12} \ , \quad (3)$$

where $S_{12} = 3(\vec{\sigma}_N \cdot \hat{r})(\vec{\sigma}_\Lambda \cdot \hat{r}) - \vec{\sigma}_N \cdot \vec{\sigma}_\Lambda$. The five $p_N s_\Lambda$ two-body matrix elements depend on the radial integrals associated with each component in Eq. (3). They are denoted by the parameters $\overline{V}$, Δ, S_Λ, S_N and T. By convention, S_Λ and S_N are actually the coefficients of $\vec{l}_N \cdot \vec{s}_\Lambda$ and $\vec{l}_N \cdot \vec{s}_N$. Then, the operators associated with Δ and S_Λ are $\vec{S}_N \cdot \vec{s}_\Lambda$ and $\vec{L}_N \cdot \vec{s}_\Lambda$. We note that $\overline{V}$ contributes only to the overall binding energy and S_N does not contribute to doublet splittings in the weak-coupling limit, but augments the nuclear spin-orbit interaction and contributes to the spacings between states based on different core states. The doublet splittings are determined to an excellent approximation by the Λ spin-dependence parameters Δ, S_Λ and T, together with important contributions from $\Lambda - \Sigma$ coupling.

The parametrization of Eq. (3) applies to the direct ΛN interaction, the ΛN–ΣN coupling interaction, and the direct ΣN interaction for both isospin $1/2$ and $3/2$. A set of parameters that fits the full particle-stable excitation spectra of $^7_\Lambda$Li and $^9_\Lambda$Be, six levels in total determined by the seven observed γ rays [14], is given by

$$\Delta = 0.430 \quad S_\Lambda = -0.015 \quad S_N = -0.390 \quad T = 0.030 \quad (A{=}7{-}9) \ , \quad (4)$$

all in MeV [15]. Somewhat reduced values were derived in the heavier p-shell hypernuclei by fitting Δ, S_Λ and T to the six Λ hypernuclear doublet splittings deduced from γ rays observed in mass $A{=}11{-}16$ hypernuclei [14], and by fixing the parameter S_N from the excitation energy of $^{16}_\Lambda$O(1^-_2):

$$\Delta = 0.330 \quad S_\Lambda = -0.015 \quad S_N = -0.350 \quad T = 0.024 \quad (A{=}11{-}16) \ , \quad (5)$$

all in MeV [15]. The corresponding matrix elements for $\Lambda - \Sigma$ coupling, based on G-matrix calculations [18] for the NSC97e,f interactions [19], are kept fixed throughout the p shell [15]:

$$\overline{V}' = 1.45 \quad \Delta' = 3.04 \quad S'_\Lambda = S'_N = -0.09 \quad T' = 0.16 \quad (\text{in MeV}). \quad (6)$$

In Table 1 we list the $\Lambda - \Sigma$ and spin-dependent contributions to g.s. separation energies B_Λ of interest in the present work, calculated in the shell

Table 1: $\Lambda-\Sigma$ and spin-dependent contributions to g.s. separation energies B_Λ used for input to the calculation of $\Lambda\Lambda$ separation energies $B_{\Lambda\Lambda}$. The spin-independent term $\overline{V}$ is obtained (see text) from B_Λ^{exp} values [20]. The B_Λ^{exp} values listed for the $A=9,10$ charge-symmetric doublets are statistical averages of the separate values, whereas the listed value for $A=12$ is $B_\Lambda^{\mathrm{exp}}(^{12}_\Lambda\mathrm{B})$.

$^A_\Lambda Z$	$^9_\Lambda\mathrm{Be}$	$^9_\Lambda\mathrm{Li}$–$^9_\Lambda\mathrm{B}$	$^{10}_\Lambda\mathrm{Be}$–$^{10}_\Lambda\mathrm{B}$	$^{11}_\Lambda\mathrm{B}$	$^{11}_\Lambda\mathrm{Be}$	$^{12}_\Lambda\mathrm{B}$–$^{12}_\Lambda\mathrm{C}$
J^π(g.s.)	$1/2^+$	$3/2^+$	1^-	$5/2^+$	$1/2^+$	1^-
$\Lambda-\Sigma$	4	183	35	66	99	103
Δ	0	350	125	203	2	108
S_Λ	0	-10	-13	-20	0	-14
S_N	207	434	386	652	540	704
T	0	-6	-15	-43	0	-29
Sum (keV)	211	952	518	858	641	869
B_Λ^{exp} (MeV)	6.71	8.44	8.94	10.24		11.37
Error	±0.04	±0.10	±0.11	±0.05		±0.06
$\overline{V}$ (MeV)	-0.84	-1.09	-1.06	-1.04		-1.05

model [15]. Subtracting these contributions plus an s_Λ single-particle energy identified with $B_\Lambda^{\mathrm{exp}}(^5_\Lambda\mathrm{He})=3.12\pm0.02$ MeV, and dividing by the number of p-nucleons $(A-5)$, we obtain the magnitude of the spin-independent $s_\Lambda p_N$ matrix element $\overline{V}$. Our error estimate for $\overline{V}$ is ±0.03 MeV. Excluding $^9_\Lambda\mathrm{Be}$ which deviates substantially from the other species, a common value of $\overline{V}^{\mathrm{SM}}=-1.06\pm0.03$ MeV emerges. The constancy of $\overline{V}$ in this mass range provides, a-posteriori, justification of the SM approach adopted here for Λ hypernuclear g.s. studies. We can immediately check whether $\overline{V}$ is globally compatible also with the SM description of $B_{\Lambda\Lambda}$ within the range of experimentally known $\Lambda\Lambda$ hypernuclei, from $^6_{\Lambda\Lambda}\mathrm{He}$ to $^{13}_{\Lambda\Lambda}\mathrm{B}$. To this end we form the difference between the $B_{\Lambda\Lambda}^{\mathrm{exp}}$ values, subtract 1.54 MeV for the contribution of $\Lambda-\Sigma$ coupling and S_N to $B_{\Lambda\Lambda}(^{13}_{\Lambda\Lambda}\mathrm{B})$ (the contributions from the Λ-spin-dependent parameters Δ, S_Λ and T average to zero in $\overline{B}_\Lambda$) and divide by $2\times(13-6)=14$ for the coefficient of $\overline{V}$ in this difference, see Eq. (2). This gives $\overline{V}^{\mathrm{SM}}=-1.06\pm0.05$ MeV, in excellent agreement with the value derived above from Λ hypernuclear systematics.

The departure of $^9_\Lambda\mathrm{Be}$ from the $\overline{V}$ systematics in Table 1 deserves discussion because $B_\Lambda(^9_\Lambda\mathrm{Be}_{\mathrm{g.s.}})$ has been a problem for SM studies of hypernuclei [17, 21]. It is a somewhat artificial problem because, by convention, $B_\Lambda(^9_\Lambda\mathrm{Be})$

and $B_{\Lambda\Lambda}(^{10}_{\Lambda\Lambda}\text{Be})$ involve the binding energy of the free ^8Be. Two α's just bound in a $2s$ relative state have a large separation. However, it takes only a few MeV of binding energy, such as the 3.5 MeV α separation energy in $^9_\Lambda\text{Be}$, for the system to be reduced to a typical p-shell size. The $^5_\Lambda\text{He}+\alpha$ system in a $1d$ relative state has a comparable radius even for a small α binding energy because of the additional centrifugal barrier. Therefore, we argue that the Λ's in $^9_\Lambda\text{Be}$ and $^{10}_{\Lambda\Lambda}\text{Be}$ interact within a nuclear core which is already of a normal p-shell size. By taking one $\overline{V}$ from $^9_\Lambda\text{Be}$ and one from the $^9_\Lambda\text{Li}$–$^9_\Lambda\text{B}$ column in Table 1, we incorporate the influence of the free ^8Be binding energy into our estimate for $B_{\Lambda\Lambda}(^{10}_{\Lambda\Lambda}\text{Be})$. In a slight variation, we can replace the contribution of the spin-independent component of the ΛN interaction to one of the $\overline{B}_\Lambda$ values in Eq. (2) by its average value $\overline{V}^{\text{SM}}$, leading to

$$B_{\Lambda\Lambda}^{\text{SM}}(^A_{\Lambda\Lambda}Z) = 2\overline{B}_\Lambda(^{A-1}_\Lambda Z) + (A-6)[\overline{V}(^{A-1}_\Lambda Z) - \overline{V}^{\text{SM}}] + <V_{\Lambda\Lambda}>_{\text{SM}}, \quad (7)$$

where $\Lambda-\Sigma$ contributions $\lesssim 0.1$ MeV are disregarded. The cluster model, on the other hand, has the freedom to treat ^8Be itself as well as the hypernuclear systems.

3. Shell-model calculations and predictions for $\Lambda\Lambda$ hypernuclei

In this section we discuss *all* the $\Lambda\Lambda$ hypernuclear species known or conjectured beyond $^6_{\Lambda\Lambda}\text{He}$, the latter serving in both the SM and CM to constrain the $\Lambda\Lambda$ interaction. These consist of the accepted $^{10}_{\Lambda\Lambda}\text{Be}$, $^{13}_{\Lambda\Lambda}\text{B}$, two KEK-E176 event interpretations that could replace the generally accepted $^{13}_{\Lambda\Lambda}\text{B}$ interpretation [16], and the recently published KEK-E373 HIDA emulsion event [4] assigned as $^{11}_{\Lambda\Lambda}\text{Be}$ or $^{12}_{\Lambda\Lambda}\text{Be}$. All are listed in Table 2. For $^{10}_{\Lambda\Lambda}\text{Be}_{\text{g.s.}}$, we follow Hiyama et al. [8] assuming that the Demachi-Yanagi event [22] corresponds to the formation of $^{10}_{\Lambda\Lambda}\text{Be}^*_{2+}$(3 MeV). The older Danysz event [23], when fitted to a π^- decay of $^{10}_{\Lambda\Lambda}\text{Be}_{\text{g.s.}}$ to $^9_\Lambda\text{Be}^*$(3 MeV), yields $B_{\Lambda\Lambda}=14.7\pm0.4$ MeV, consistent with the value listed in the table. For $^{13}_{\Lambda\Lambda}\text{B}$, observed in KEK-E176 [24], we follow the recent E4 event classification in Ref. [16] assuming a π^- decay of $^{13}_{\Lambda\Lambda}\text{B}_{\text{g.s.}}$ to $^{13}_\Lambda\text{C}^*$(4.9 MeV). These particular identifications for $^{10}_{\Lambda\Lambda}\text{Be}$ and $^{13}_{\Lambda\Lambda}\text{B}$, ensure that $0 \leq \Delta B_{\Lambda\Lambda} \lesssim 1$ MeV in both, similarly to Eq. (1) for $^6_{\Lambda\Lambda}\text{He}$.

$B_{\Lambda\Lambda}^{\text{SM}}(^A_{\Lambda\Lambda}Z)$ predictions obtained by applying Eq. (2) [Eq. (7) for $^{10}_{\Lambda\Lambda}\text{Be}$] are listed in Table 2 together with the $\overline{B}_\Lambda(^{A-1}_\Lambda Z)$ input which is constrained by known $B_\Lambda^{\text{exp}}(^{A-1}_\Lambda Z)$ values [20]. A brief discussion is due.

Table 2: Comparison between $B_{\Lambda\Lambda}^{\exp}$ from KEK-E176,E373 and $B_{\Lambda\Lambda}^{\mathrm{SM}}(_{\Lambda\Lambda}^{A}Z)$ predictions using Eq. (2) [Eq. (7) for $_{\Lambda\Lambda}^{10}$Be]. Input $\overline{B}_\Lambda$ values, as well as $B_{\Lambda\Lambda}^{\mathrm{CM}}(_{\Lambda\Lambda}^{A}Z)$ values [13] where available, are also listed. All values are in MeV. The $B_{\Lambda\Lambda}^{\exp}$ values from KEK-E176 [16] refer to different interpretations of the same emulsion event.

$_{\Lambda\Lambda}^{A}Z$	$\overline{B}_\Lambda(_{\Lambda}^{A-1}Z)$	$B_{\Lambda\Lambda}^{\mathrm{SM}}(_{\Lambda\Lambda}^{A}Z)$	$B_{\Lambda\Lambda}^{\exp}(_{\Lambda\Lambda}^{A}Z)$	$B_{\Lambda\Lambda}^{\mathrm{CM}}(_{\Lambda\Lambda}^{A}Z)$
$_{\Lambda\Lambda}^{6}$He	3.12 ± 0.02	6.91 ± 0.16	6.91 ± 0.16 [4]	6.91
$_{\Lambda\Lambda}^{10}$Be	6.71 ± 0.04	14.97 ± 0.22	14.94 ± 0.13 [22]	14.74
$_{\Lambda\Lambda}^{11}$Be	8.86 ± 0.11	18.40 ± 0.28	17.53 ± 0.71 [16]	18.23
			20.83 ± 1.27 [4]	
$_{\Lambda\Lambda}^{12}$Be	10.02 ± 0.05	20.72 ± 0.20	22.48 ± 1.21 [4]	–
$_{\Lambda\Lambda}^{12}$B	10.09 ± 0.05	20.85 ± 0.20	20.60 ± 0.74 [16]	–
$_{\Lambda\Lambda}^{13}$B	11.27 ± 0.06	23.21 ± 0.21	23.3 ± 0.7 [16]	–

- In the $B_{\Lambda\Lambda}(_{\Lambda\Lambda}^{10}Be)$ calculation, to account for the loose structure of the ^{8}Be core, we used Eq. (7) with $\overline{V}^{\mathrm{SM}} = -1.06 \pm 0.03$ MeV. The calculated $B_{\Lambda\Lambda}^{\mathrm{SM}}$ provides excellent agreement within errors with the experimental value, and departs within $\approx 1\sigma$ from the CM calculated value. While Eq. (7) uses the p-shell *average* ΛN matrix element $\overline{V}^{\mathrm{SM}}$, it is possible alternatively to proceed locally focusing on the $A = 9$ Λ hypernuclei. In this procedure we replace one of the two $\overline{B}_\Lambda(_{\Lambda}^{9}Be)$ in Eq. (2) by an appropriate contribution from the more compact $_{\Lambda}^{9}$Li and $_{\Lambda}^{9}$B hypernuclei. Specifically, we subtract the 'sum' entry of 952 keV in Table 1 from $B_{\Lambda}^{\exp}(_{\Lambda}^{9}Li - _{\Lambda}^{9}B) = 8.44$ MeV there, adding the proper 'sum' entry of 211 keV for $_{\Lambda}^{9}$Be, which results in 7.70 MeV. The appropriate $\overline{B}_\Lambda$ is then given by the average of 6.71 MeV for the first Λ, as in $_{\Lambda}^{9}$Be, and 7.70 MeV for the second Λ. This procedure results in $B_{\Lambda\Lambda}^{\mathrm{SM}}(_{\Lambda\Lambda}^{10}Be) = 15.08 \pm 0.20$ MeV, in agreement with $B_{\Lambda\Lambda}^{\exp}(_{\Lambda\Lambda}^{10}Be)$ within error bars.

- In the $B_{\Lambda\Lambda}(_{\Lambda\Lambda}^{11}Be)$ calculation, $\overline{B}_\Lambda(_{\Lambda}^{10}Be)$ was derived from a 120 keV g.s. doublet splitting provided by the ΛN interaction parameters (5) and (6). The corresponding $2^- \to 1^- \gamma$ ray transition has not been observed. One reason could be that the splitting is considerably smaller, causing the excited state to undergo weak decay. In the extreme case of degenerate doublet levels, using $\overline{B}_\Lambda = B_\Lambda^{\exp}(_{\Lambda}^{10}Z_{\mathrm{g.s.}}) = 8.94$ MeV, the calculated $B_{\Lambda\Lambda}^{\mathrm{SM}}(_{\Lambda\Lambda}^{11}Be)$ would increase by 0.15 MeV with respect to the

value listed in the table. We note that the SM prediction and the CM prediction agree with each other within error bars in spite of different ΛN interaction inputs. This agreement might be fortuitous. Of the two experimental $B_{\Lambda\Lambda}$ assignments, only the KEK-E176 [16] G2 alternative assignment to $^{13}_{\Lambda\Lambda}\text{B}$ is in rough agreement within errors with the theoretical predictions.

- In the $B_{\Lambda\Lambda}(^{12}_{\Lambda\Lambda}\text{Be})$ calculation, we subtracted the specific 'sum' of 858 keV in Table 1 from $B_{\Lambda}^{\text{exp}}(^{11}_{\Lambda}\text{B}) = 10.24$ MeV there, adding the proper 'sum' of 641 keV for $^{11}_{\Lambda}\text{Be}_{\text{g.s.}}$ to obtain $\overline{B}_{\Lambda}(^{11}_{\Lambda}\text{Be})$. The calculated $B_{\Lambda\Lambda}^{\text{SM}}(^{12}_{\Lambda\Lambda}\text{Be})$ disagrees with a KEK-E373 Hida event assignment.

- In the $B_{\Lambda\Lambda}(^{12}_{\Lambda\Lambda}\text{B})$ calculation, $\overline{B}_{\Lambda}(^{11}_{\Lambda}\text{B})$ was derived from the observed 263 keV g.s. doublet splitting [25]. The predicted $B_{\Lambda\Lambda}$ value is in good agreement with the KEK-E176 [16] G3 alternative assignment to $B_{\Lambda\Lambda}(^{13}_{\Lambda\Lambda}\text{B})$.

- In the $B_{\Lambda\Lambda}(^{13}_{\Lambda\Lambda}\text{B})$ calculation, since the value of $B_{\Lambda}^{\text{exp}}(^{12}_{\Lambda}\text{C}_{\text{g.s.}})$ is based on only 6 events and is unsettled, we used the much better determined $B_{\Lambda}^{\text{exp}}(^{12}_{\Lambda}\text{B}_{\text{g.s.}})$ [20] plus a 161 keV g.s. doublet splitting from the observed $2^- \rightarrow 1^-$ γ ray transition in the charge-symmetric hypernucleus $^{12}_{\Lambda}\text{C}$ [25].

The very good agreement in Table 2 between $B_{\Lambda\Lambda}^{\text{SM}}(^{10}_{\Lambda\Lambda}\text{Be})$ and $B_{\Lambda\Lambda}^{\text{exp}}(^{10}_{\Lambda\Lambda}\text{Be})$, and between $B_{\Lambda\Lambda}^{\text{SM}}(^{13}_{\Lambda\Lambda}\text{B})$ and $B_{\Lambda\Lambda}^{\text{exp}}(^{13}_{\Lambda\Lambda}\text{B})$, provides a consistency check from below and from above on the predicted values in between. Our calculations suggest that the KEK-E176 [16] event interpretations G2 for $^{11}_{\Lambda\Lambda}\text{Be}$ and G3 for $^{12}_{\Lambda\Lambda}\text{B}$, with $B_{\Lambda\Lambda}$ values listed in Table 2, cannot be excluded.[2] In contrast, it is difficult to reconcile the KEK-E373 Hida event interpretation as either $^{11}_{\Lambda\Lambda}\text{Be}$ or $^{12}_{\Lambda\Lambda}\text{Be}$ [4] with the calculated $B_{\Lambda\Lambda}^{\text{SM}}$ listed in the table. We conclude that while a $^{12}_{\Lambda\Lambda}\text{Be}$ assignment of the HIDA event is somewhat more likely than a $^{11}_{\Lambda\Lambda}\text{Be}$ assignment, both assignments are ruled out by the SM.

4. Discussion and Conclusion

Historically, cluster models charted the $B_{\Lambda\Lambda}$ map of $\Lambda\Lambda$ hypernuclei in the p shell [6, 7, 8, 13, 26]. However, the CM has not gone, for obvious

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[2] An alternative production reaction to G3, specified by E2 and resulting in $B_{\Lambda\Lambda}(^{12}_{\Lambda\Lambda}\text{B}) = 20.02 \pm 0.78$ MeV [16], also admits a $^{12}_{\Lambda\Lambda}\text{B}$ assignment.

computational reasons, beyond $^{11}_{\Lambda\Lambda}$Be. Furthermore, the CM has not been able to incorporate the full range of Λ spin-dependent interactions that enter hypernuclear computations. The shell model provides a viable alternative [15, 17, 21]. In this work we demonstrated that the simple SM estimate Eq. (2) for $\Lambda\Lambda$ separation energies $B_{\Lambda\Lambda}$ in $\Lambda\Lambda$ hypernuclei, in terms of $(2J+1)$ averaged g.s. doublet separation energies $\overline{B}_\Lambda$ in Λ hypernuclei, works well for the known $\Lambda\Lambda$ hypernuclear species. The spectroscopic information required to devise the $\overline{B}_\Lambda$ input that dominates the estimate for $B_{\Lambda\Lambda}$ is now available through recent SM studies [15] that derive the spin-dependent ΛN interaction parameters from the observed γ ray transitions in p-shell Λ hypernuclei [14]. We estimate the precision of $B_{\Lambda\Lambda}$ values thus extracted to be about 0.2 MeV.

It was acknowledged that the application of the SM to Λ hypernuclei throughout the p shell suffers from inability to reproduce correctly the g.s. Λ separation energy in $^9_\Lambda$Be because of the loose structure of its particle unstable ^{8}Be core. We have indicated a way to bypass this difficulty in $^{10}_{\Lambda\Lambda}$Be by using an alternative SM estimate, Eq. (7), that restores the desired level of predictability to the SM in this particular case. Using the uniquely assigned $^6_{\Lambda\Lambda}$He datum, we were able to derive good estimates for the $\Lambda\Lambda$ separation energies of the other two known species $^{10}_{\Lambda\Lambda}$Be and $^{13}_{\Lambda\Lambda}$B in terms of experimentally derived Λ hypernuclear separation energies, augmented by SM p-shell systematics. Our predictions for $^{11}_{\Lambda\Lambda}$Be, $^{12}_{\Lambda\Lambda}$Be and $^{12}_{\Lambda\Lambda}$B suggest that whereas a $^{11}_{\Lambda\Lambda}$Be or $^{12}_{\Lambda\Lambda}$B interpretation for the KEK-E176 emulsion event [16] generally accepted as $^{13}_{\Lambda\Lambda}$B cannot be excluded, the recently reported KEK-E373 HIDA event [4] is unlikely to fit a proper $^{11}_{\Lambda\Lambda}$Be or $^{12}_{\Lambda\Lambda}$Be assignment.

Acknowledgements

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